Scaling Properties of Price Changes for Korean Stock Indices

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We consider returns of two Korean stock market indices, KOSPI and KOSDAQ index. Central parts of the probability distribution function of returns are well fitted by the Lorentzian distribution function. However, tail parts of the probability distribution function follow a power law behavior well. We found that the probability distribution function of returns for both KOSPI and KOSDAQ, is outside the Lévy stable distribution.

I. INTRODUCTION

Complex behaviors of econophysics have greatly led to attentions in the field of statistical physics. A lot of economic data have been reanalyzed by physicists recently[1, 2, 3, 4]. Time series of stock market around the world have rich behaviors. The time series deviate from the EMH(efficient market hypethesis). Indices of stock market show scaling behaviors in the well developed market. It is very difficult to understand the dynamics of financial systems because there are many factors among interacting agents.

Bachelier has proposed a financial model of stochastic process of returns which consider the variation of share prices as an independently, identically distributed (i.i.d) Gaussian random variable[5]. However, the distribution of returns in financial markets does not follow Gaussian distribution. Mandelbrot analyzed a relatively short time series of cotton prices and observed that returns have Lévy stable symmetric distribution with Pareto fat tail[6, 7, 8, 9]. Boston group reported departures from Lévy stable distribution of returns by analyzing high frequency data points of the S & P 500 index[10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21]. They observed that large events was very frequent in the data, a fact largely underestimated by a Gaussian process. They also found a power-law behavior of the probability density function (pdf) of returns with the fat tail exceeding the Lévy stable distribution. The similar behavior reported for distribution of returns of other indices, including FOREX[22], DAX[23], and Hang-Seng indices[24]

In this article we consider two Korean stock market indices, KOSPI and KOSDAQ index. We consider a set of data recorded per one second for KOSPI from March 30 1992 to November 30 1999 and for KOSDAQ from March 5 2001 to February 28 2003. KOSDAQ index is recorded in 30 seconds interval. We count the time during trading hours and remove closing hours, weekends and holidays from data sets. For a time series $Z(t)$ of stock market index values, the return $G_T(t)$ over a return time T is defined as

$$G_T(t) = \ln \frac{Z(t+T)}{Z(t)} \tag{1}$$

For small changes in $Z(t)$, the return is approximately

$$G_T(t) \simeq \frac{Z(t+T) - Z(t)}{Z(t)} \tag{2}$$

The normalized return is defined as

$$g_T(t) = \frac{G_T(t) - < G_T(t) >}{\sigma(G_T(t))} \tag{3}$$

where $\sigma(G)$ is the standard deviation and $< \cdots >$ denotes averaging over time variable.

FIG. 1: The normalized return versus time for KOSPI

FIG. 2: The normalized return versus time for KOSDAQ.

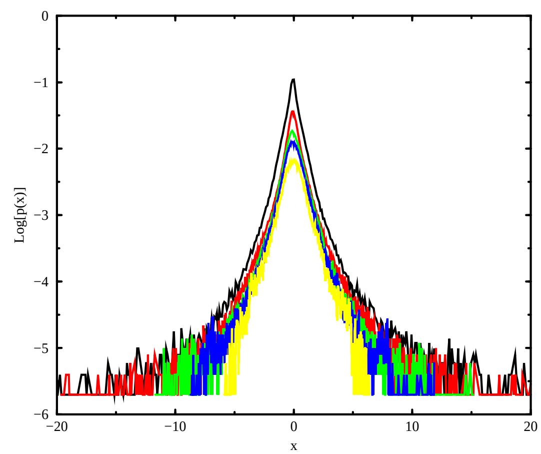

FIG. 3: The semilogarithmic plot of the probability distribution function as a function of the return with the return time T=1min(top), 10min, 30min, 60min and 600min(bottom) for KOSPI.

II. RESULTS AND DISCUSSIONS

The normalized return presented in Fig.1 for KOSPI and in Fig.2 for KOSDAQ. We observed large price

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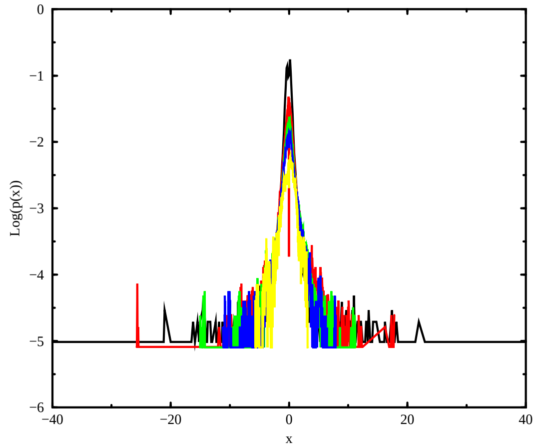

FIG. 4: The semilogarithmic plot of the probability distribution function as a function of the return with the return time T=1min(top), 10min, 30min, 60min and 600min(bottom) for KOSDAQ.

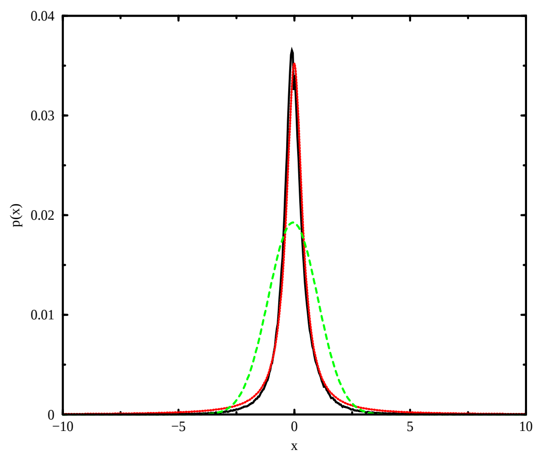

FIG. 5: Fitting of the probability distribution function with the return time T=10min by the Gaussian function (dashed line) and by the Lorentzian (dotted line) function for KOSPI.

changes around the period of Asian economic crisis in September 1997. We consider the logarithmic return with the return time T=1min, 10min, 30min, 60min, and 600min. The pdf for the return presented in Fig.3 (KOSPI) and Fig.4 (KOSDAQ). For the short return time T=1min, the pdf of the return has long tails with very large fluctuations. The peak of the pdf of the return decreases as the return time T increases. We fit the pdf with the Gaussian distribution function and Lorentzian distribution function for the return time T=10min in Fig.5(KOSPI) and Fig.6(KOSDAQ). The central region of the pdf is fitted better by Lorentzian than by Gaussian[25]. However, the positive and negative tail region of the pdf deviates from Gaussian and Lorentzian. The tail of the pdf of returns decays according to a power law as

$$p(x) \sim x^{-(1+\alpha)} \tag{4}$$

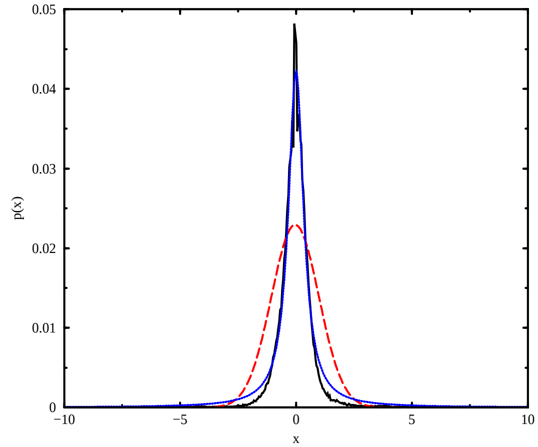

FIG. 6: Fitting of the probability distribution function with the return time T=10min by the Gaussian function (dashed line) and by the Lorentzian (dotted line) function for KOSDAQ.

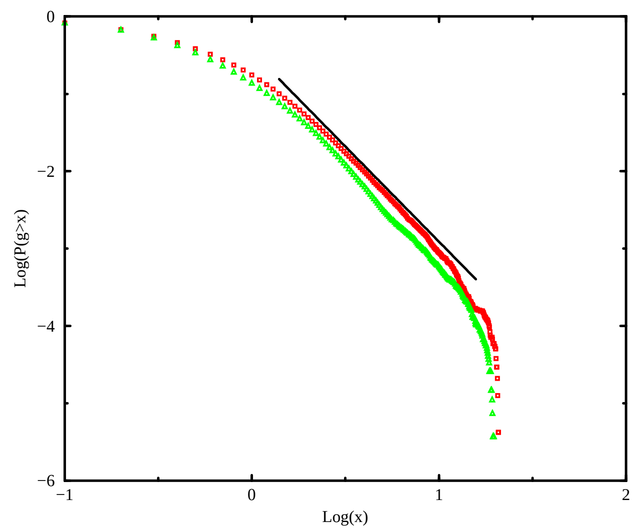

FIG. 7: The log-log plot of the accumulated probability distribution function as a function of the return with the return time T=10min for KOSPI.

with the exponent $\alpha > 2$. The accumulated pdf of returns is defined as

$$P(g > x) = \int_x^\infty p(x)dx \tag{5}$$

The accumulated pdf follows a power-law behavior as

$$P(g > x) \sim \frac{1}{x^\alpha} \tag{6}$$

In Fig.7(KOSPI) and Fig.8(KOSDAQ), we presented the accumulated probability distribution function for the return time T=10min. We observed that the exponents α are greater than 2 which means that the pdf of returns deviated from the stable Lévy distribution with $0 < \alpha < 2$. We present the exponents α for the different

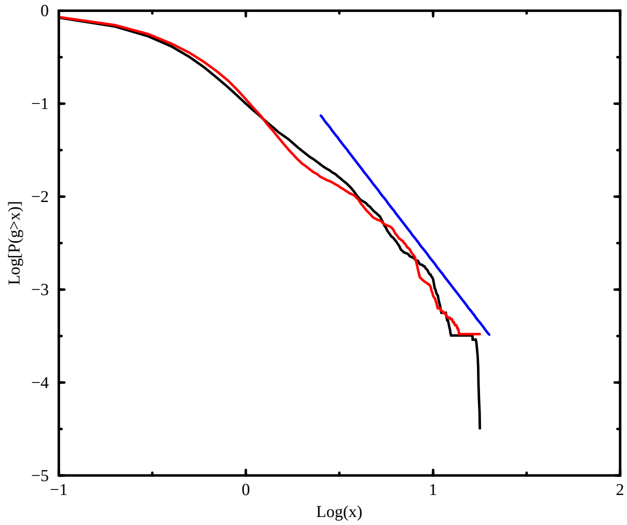

FIG. 8: The log-log plot of the accumulated probability distribution function as a function of the return with the return time T=10min for KOSDAQ.

return time T in the table 1. The exponent α was measured for different stock indices over many countries. We summarized the measured exponent α in the table 1 for many different stock indices. Exponents α of KOSPI and KOSDAQ increase when the return time increases. We also observed that the range of the power-law diminish as the return time increases. The pdf of the return deviated from the stable Lévy distribution for all different stock indices. The exponents α of the positive and negative tail are greater than 2. The exponent α of the positive tail of KOSPI and KOSDAQ are sightly less than one of the negative

From the analysis of the stock market indices, we observed that the pdf of KOSPI index is well outside the stable Lévy distribution. The exponent α depends on the return time T. The observed exponent α increased when the return time increased for both positive and negative tail. The pdf of KOSDAQ index also is outside the stable Lévy distribution with $\alpha > 2$ for the positive tail.

III. CONCLUSIONS

We consider the probability density function(pdf) of the Korean stock market indices, KOSPI and KOSDAQ indices. We observe that the pdf for both KOSPI and KOSDAQ indices fit neither Gaussian nor Lorentzian distribution function. Central parts of indices are well fitted by the Lorentzian distribution function. However, the tail parts of the pdf deviate strongly form Gaussian and Lorentzian distribution function. The tail part of the pdf follows a power-law asymptotic behavior. We observe that exponents of the power-law of the accumulated probability distribution function are well outside the stable Lévy distribution. Korean stock market is not

TABLE I: Summary of exponents α for the pdf for the positive tail

	positive tail		
	α	range	Ref
	2.16 (T=1min)	$1 < g < 12$	[1]
	2.46 (T=10min)	$1.4 < g < 10$	[1]
KOSPI	2.71 (T=30min)	$1.5 < g < 10$	[1]
	2.87 (T=60min)	$2 < g < 7$	[1]
	2.87 (T=600min)	$2 < g < 4$	[1]
	2.06 (T=1min)	$10 < g < 30$	[1]
	2.22 (T=10min)	$3 < g < 10$	[1]
KOSDAQ	2.39 (T=30min)	$1.6 < g < 6.3$	[1]
	2.70 (T=60min)	$1.6 < g < 4$	[1]
	2.46 (T=600min)	$1 < g < 2.2$	[1]
	2.4 (T=1min)	$2 < g < 20$	[2]
DAX	2.9 (T=10min)		[2]
	3.5 (T=60min)		[2]
	3.5 (T=1day)		[2]
	2.32 (T=1min)	$3 < g < 15$	[3]
Hang-Seng	3.05 (T=1day)	$1 < g$	[4]
	5.0 (T=1day)		[3]
	2.95 (T=1min)	$3 < g < 50$	[4]
	3.45 (T=1min)		[4]
	2.69 (T=16min)		[4]
S & P 500	2.53 (T=32min)		[4]
	2.83 (T=128min)		[4]
	3.39 (T=512min)		[4]
	3.34 (T=1day)		[4]
Nikkei	3.05 (T=1day)	$1 < g$	[4]

TABLE II: Summary of exponents α for the pdf for the negative tail

	negative tail		
	α	range	Ref
	2.29 (T=1min)	$1 < g < 12$	[1]
	2.56 (T=10min)	$1.4 < g < 10$	[1]
KOSPI	2.73 (T=30min)	$1.5 < g < 8$	[1]
	3.03 (T=60min)	$2 < g < 7$	[1]
	2.91 (T=600min)	$2 < g < 3.4$	[1]
	2.41 (T=1min)	$10 < g < 30$	[1]
	2.62 (T=10min)	$3 < g < 12$	[1]
KOSDAQ	1.89 (T=30min)	$1.6 < g < 6.3$	[1]
	2.09 (T=60min)	$1.6 < g < 6.3$	[1]
	1.88 (T=600min)	$1.2 < g < 2.5$	[1]
DAX	2.6(T=1min)	$2 < g < 20$	[2]
	2.32 (T=1min)	$3 < g < 15$	[3]
Hang-Seng	4.0 (T=1day)		[3]
	2.75 (T=1min)	$3 < g < 50$	[4]
Hang-Seng	3.29 (T=1min)	Hill estimator	[4]

[1] Present work, [2] Ref.16, [3] Ref.17, [4] Ref.10

described by the random Gaussian stochastic processes.

Acknowledgments

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